For the converse, assume that AY and BY are independent. Then, $\forall u, v$,

$$M_W(\theta) = M_{AY}(u) M_{BY}(v), \quad \forall u, v,$$

$$\implies \exp\left\{\frac{1}{2}\left(u^\top A\Omega A^\top u + v^\top B\Omega B^\top v + u^\top A\Omega B^\top v + v^\top B\Omega A^\top u\right)\right\}$$

$$= \exp\left\{\frac{1}{2}u^\top A\Omega A^\top u\right\} \exp\left\{\frac{1}{2}v^\top B\Omega B^\top v\right\}$$

$$\implies \exp\left\{\frac{1}{2} \times 2u^\top A\Omega B^\top v\right\} = 1$$

$$\implies u^\top A\Omega B^\top v = 0, \qquad \forall\ u \in \mathbb{R}^d, v \in \mathbb{R}^m,$$

$\implies$ the orthocomplement[a] of the column space of $A\Omega B^\top$ is the whole of $\mathbb{R}^m$.

$$\implies A\Omega B^\top = 0.$$

$\square$

[a] recall that for $Q_{m\times d}$ we have $\mathcal{M}^\perp(Q) = \{y \in \mathbb{R}^m : y^\top Qx = 0,\ \forall x \in \mathbb{R}^d\}$